Proof by Incomplete Enumeration
and Other Logical Misconceptions

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Overview

• We’re studying student misconceptions in logic design
  1. Improve instruction
  2. Build reliable tests of student understanding

• Motivation
• What we’ve done
• Some of what we’ve learned
Goal of education research

• Goal: Improving the way we teach

• State of the art: completely unscientific
  “I tried ____ and the students liked it”
  “I tried ____ and I thought it worked”

• What matters is how much is learned
  – Need a “learn-o-meter”
One Success: FCI

- **Force Concept Inventory (FCI)**
  - Developed by faculty at Arizona State
  - Based on observations that students were failing to think Newtonian
    - Even when they could solve quantitative problems
  - Conceptual questions (no calculation req’d)
  - Multiple Choice
  - Distractors come from student misconceptions
Example FCI problem

FCI Question #12 (Cannon)

A ball is fired by a cannon from the top of a cliff as shown below. Which of the paths 1-5 would the cannon ball most closely follow?

Reliability

- Turns out to be pretty reliable
  - Questions trivial for Newtonian thinkers
  - Difficult for non-Newtonian thinkers
    - Distractors based on common misconceptions

- Can use it to do science:
  - Comparative Pedagogy

- Helps dissemination of best practices
Comparative Pedagogy: Hake

Interactive-engagement vs. Traditional Methods: A six-thousand student survey of mechanics test data for introductory physics courses

Richard R. Hake

Finding: Interactive engagement gives roughly twice the benefit of lecture.
– 62 introductory physics courses
Physics Education Revolution

• The FCI is significantly responsible for starting a revolution in Physics Ed.

• Can we replicate this in CS and CE?
  – We’re sure as hell going to try….
    • Starting with intro courses: Discrete Math, Programming Fundamentals (CS1), Logic Design
Step 1

- **Identify topics for logic design CI**
  - representative topics; need not be exhaustive
  - topics should be “important” and “difficult”

- For widespread use, want consensus
  - Survey outside experts

Delphi Process

• Identify Experts:
  ~20 textbook authors, pedagogical researchers
  – Selected from a diverse set of institutions

• 4 phase process
  – Conducted anonymously
    • To prevent reputation from swaying people
Phase 1: Identifying the Concepts

- Experts suggested wide range of concepts
- 2-3 researchers independently coded, clustered suggestions
- Researchers reconciled shortened lists

Number Representations: Understanding the relationship between representation (pattern) and meaning (value) (e.g., two’s complement, signed vs. unsigned, etc.)
Phases 2-4: Rating Process

**Experts’ Tasks**
- Suggested 10-15 most important, difficult concepts
- Rated concepts’ importance, difficulty on 1-10 scale
- Rated concepts, explained ratings outside intervals
- Read explanations, rated concepts

**Researchers’ Tasks**
- Synthesized List of Concepts
- Computed ratings’ mean, std. dev., intervals
- Same computations, anonymized explanations
- Same computations, determined final rankings
What we found

- An “important and difficult” subset
Digital Logic Consensus

- Understand, manipulate state in different contexts
  - 4 of 11 most important and difficult
- Divide and conquer design problems
  - 4 of 11 most important and difficult

**Top 11 Most Important and Difficult Concepts**

- Converting Verbal Specs to State Diagrams
- Timing Diagrams to State Machines
- Debugging, Troubleshooting and Designing Simulations
- Converting Algorithms to RTL
- State Transitions
- Converting Verbal Specs to Boolean Logic
- Designing Control for Datapaths
- Modular Design
- Multilevel Synthesis
- Hierarchical Design
- Sequential Circuit Corresponds to State Diagram
Step 2

• **Identify Student Misconceptions**
  – Go to the source: the students
Methodology

• Have students solve difficult problems
  – While verbalizing their thought process
  – 7 B/C-level students (2 women/5 men, 2 inter)

• Record / Transcribe

• Qualitative Analysis:
  – Researchers “code” interviews independently
  – Reconcile coding
  – Identify themes
Lessons Learned (so far)

• A sampling from two publications:

Student Misconceptions in an Introductory Logic Design Course [ASEE 2006]

Proof by Incomplete Enumeration and Other Logical Misconceptions [ICER 2008]
Converting English to Boolean

A campus sandwich shop has the following rules for making a good sandwich:

(1) \( h + r + t \) (OR) must have at least one type of meat,
(2) \( hr' + rh' \) (XOR) must have roast beef or ham, but not both,
(3) \( t' + c \) (implication) has turkey then it must also have cheese.

Write a Boolean expression for the allowed combinations of sandwich ingredients using the following variables:

\[ c = \text{cheese} \quad h = \text{ham} \quad r = \text{roast beef} \quad t = \text{turkey} \]

\[ (h + r + t)(hr' + rh')(t' + c) \rightarrow (hr' + rh')(t' + c) \]
Easy and Hard Operators

• Easy operators: AND, OR, XOR
  – Correct routinely, could explain why

• Hard operators: if-then (implication), if-and-only-if (XNOR), not both (NAND)
  – Frequently incorrect, false reasoning
  – Failure to draw truth tables
  – Often “simplified” to easier concepts
Reduction to Easy Operators

- Proof-by-incomplete enumeration
- Forgetting negated cases

<table>
<thead>
<tr>
<th>inputs</th>
<th>easy operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
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<tr>
<td>1</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Examples

INTERVIEWER: So how'd you come up with (a'c OR ac') for "do not use both?"

STUDENT 2: Well, when we do not have all spice, I mean it says do not use all spice and nutmeg simultaneously (<a,c> = <1,1>), right?

INTERVIEWER: Ok.

STUDENT 2: So if allspice is not being used, we can use cinnamon (<a,c> = <0,1>). And if allspice is used, then we cannot use cinnamon (<a,c> = <1,0>).

STUDENT 5: If you have turkey, then you must also have cheese [writes +tc] so it's turkey AND cheese

STUDENT 3: And then [rule] 3... I guess would just be like, turkey implies cheese, so let's see... turkey AND cheese (<t,c> = <1,1>) because... OR.. NOT turkey AND cheese (<t,c> = <0,1>)? I think, because this would be such true, if it has turkey and cheese, but it doesn't say anywhere that cheese cannot be by itself. So this can also be true. [writes tc + t'c].
Omission of Complemented Variables

- Failed to enumerate cases (as above)
- Left them out of expressions:
  - A sandwich with just cheese written as \(c\)
    - Instead of ch’t’r’

STUDENT 5: You can use cinnamon by itself without the nutmeg, because that doesn't break rule (2) [writes +c]
… or you could just use allspice by itself [writes +a].
Others

• Lack of meta-cognition
  – Never return to original specification

• Cowboy composition & non-systematic approaches

• If-then misconceptions
  – False Antecedent confusion

• Recall vs. reasoning
State Encoding

A state diagram with \( n \) states requires at least \( m \) flip-flops to implement a sequential circuit. If a different state diagram has \( 2n \) states, what is the minimum number of flip-flops needed for an implementation?

a.) \( m \)
b.) \( m + 1 \)
c.) \( 2m \)
d.) \( 2m + 1 \)
e.) \( m^2 \)
f.) \( m^2 + 1 \)
g.) None of the above

• 65% correct
Gate-level Reasoning

Which of the following will result in nontrivial output (not always 0 or 1)?
Select all correct answers.

a) ![Diagram a]

b) ![Diagram b]

c) ![Diagram c]

d) ![Diagram d]

e) ![Diagram e]

f) ![Diagram f]

g.) None of the above

• 59% correct
Logical Completeness

Which of the following are complete logic families (i.e., all possible Boolean functions can be implemented using just these gates and the constants 0 and 1).

Select all correct answers.

- a)
- b)
- c)
- d)
- e)
- f) None of the above

• 16% correct
Future Steps

• Step 3: Develop Concept Inventory
• Step 4: Validate
• Step 5: Disseminate
Summary

• Significant steps forward in teaching requires scientific approach to pedagogy
  – Develop tools to measure learning

• Our efforts based on previous success in another discipline (Physics)

• Big project, making steady progress
  – But, our findings can guide teaching now

http://www-faculty.cs.uiuc.edu/~zilles/csci.html