Proof by Incomplete Enumeration
and Other Logical Misconceptions

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Overview

- **We're studying student misconceptions in logic design**
  1. Improve instruction
  2. Build reliable tests of student understanding
- **Motivation**
- **What we've done**
- **Some of what we've learned**

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Goal of education research

- **Goal: Improving the way we teach**
- State of the art: **completely unscientific**
  "I tried _____ and the students liked it"
  "I tried _____ and I thought it worked"
- **What matters is how much is learned**
  - Need a "learn-o-meter"

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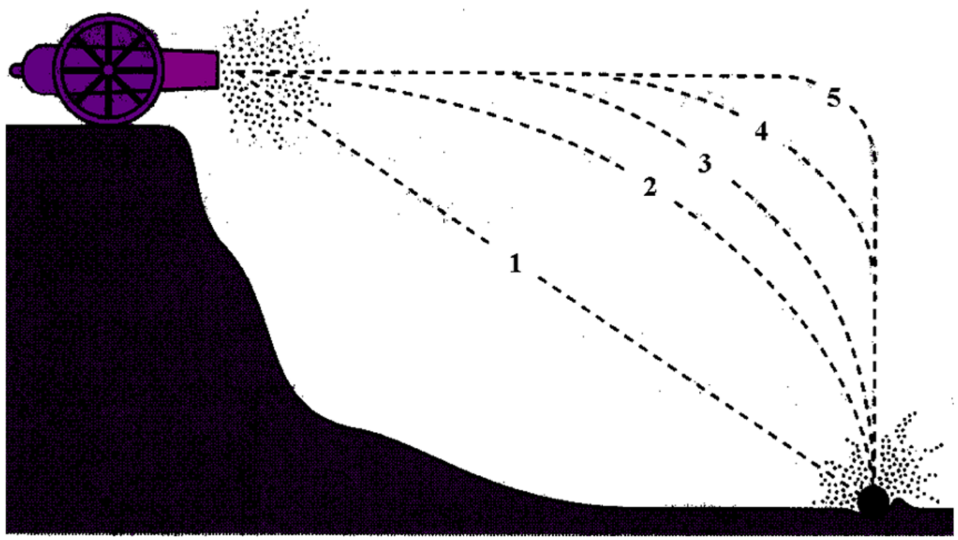
One Success: FCI

- **Force Concept Inventory (FCI)**
  - Developed by faculty at Arizona State
  - Based on observations that students were failing to think Newtonian
    - Even when they could solve quantitative problems
  - Conceptual questions (no calculation req'd)
  - Multiple Choice
  - Distractors come from student misconceptions

Example FCI problem

FCI Question #12 (Cannon)

A ball is fired by a cannon from the top of a cliff as shown below. Which of the paths 1-5 would the cannon ball most closely follow?



Mestre, Jose P. (2005). Facts and myths about pedagogies of engagement in science learning. *Peer Review, 7(2)*.

Reliability

- **Turns out to be pretty reliable**
  - Questions trivial for Newtonian thinkers
  - Difficult for non-Newtonian thinkers
    - Distractors based on common misconceptions
- **Can use it to do science:**
  - Comparative Pedagogy
- **Helps dissemination of best practices**

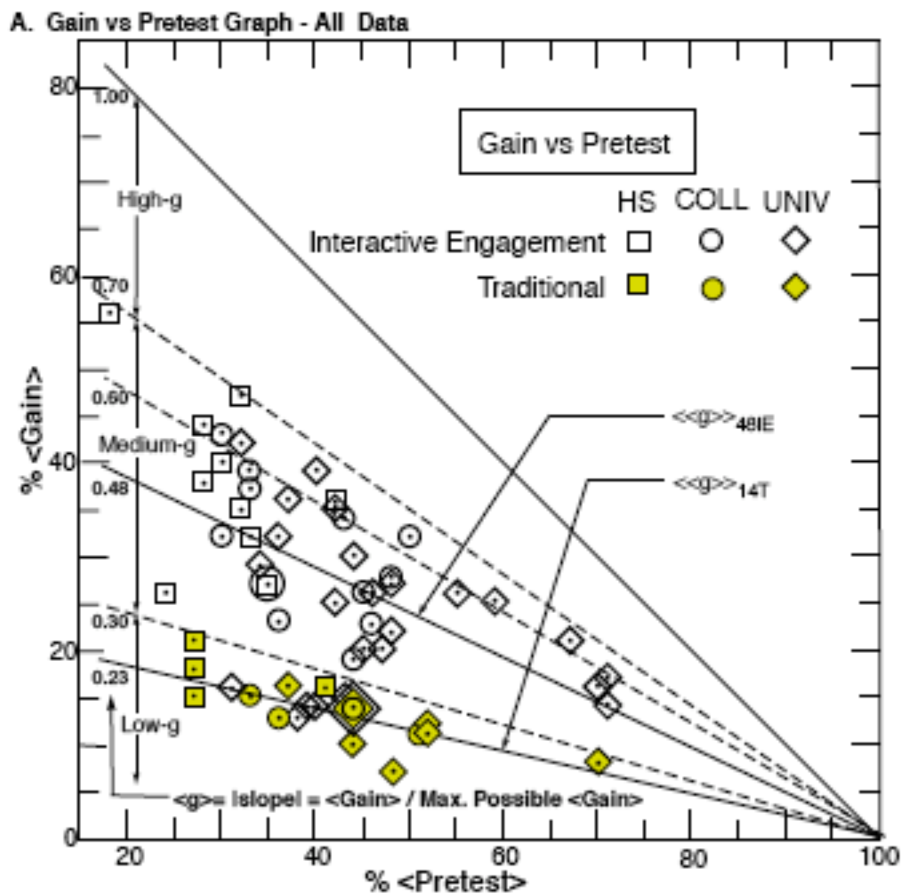
Comparative Pedagogy: Hake

Interactive-engagement vs. Traditional Methods: A six-thousand student survey of mechanics test data for introductory physics courses

Richard R. Hake

Finding: Interactive engagement gives roughly twice the benefit of lecture.

– 62 introductory physics courses



Physics Education Revolution

- **The FCI is significantly responsible for starting a revolution in Physics Ed.**

- **Can we replicate this in CS and CE?**
  - We’re sure as hell going to try….
    - Starting with intro courses: Discrete Math, Programming Fundamentals (CS1), Logic Design

Step 1

- **Identify topics for logic design CI**
  - representative topics; need not be exhaustive
  - topics should be “important” and “difficult”

  - For widespread use, want consensus
    - Survey outside experts

Identifying Important and Difficult Concepts in Introductory Computing Courses using a Delphi Process, SIGCSE 2008

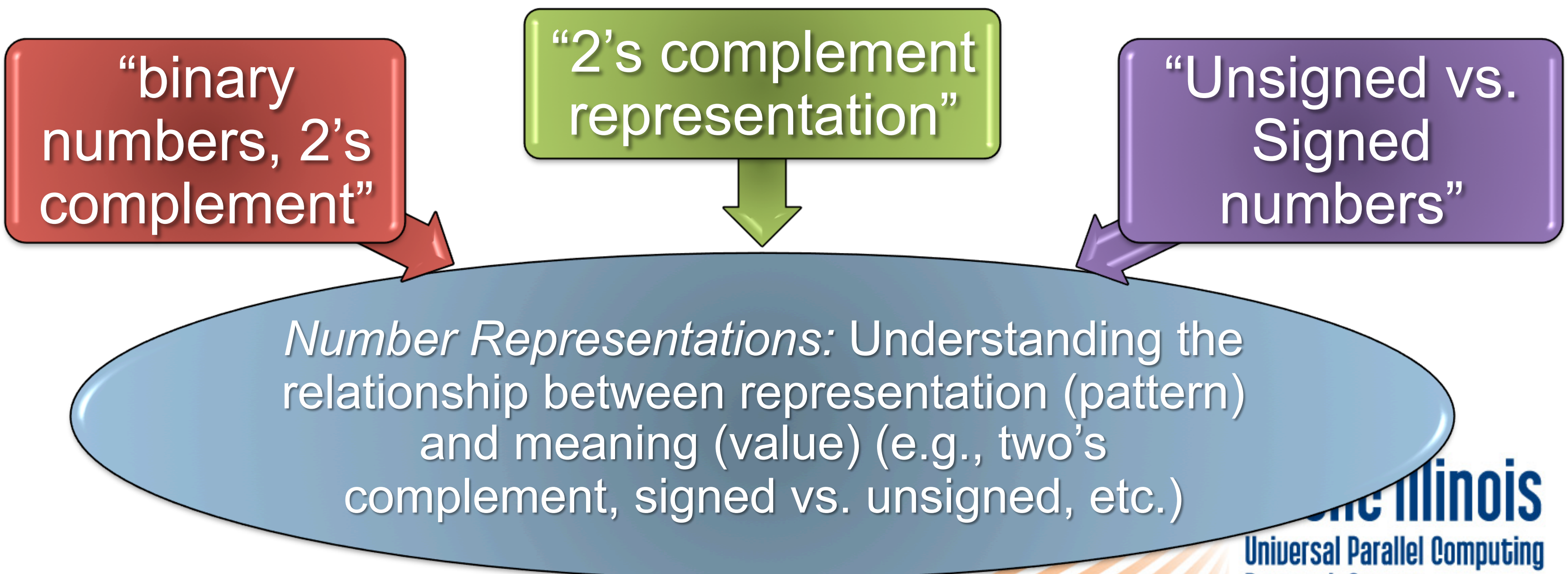
Delphi Process

- **Identify Experts:**
  ~20 textbook authors, pedagogical researchers
  – Selected from a diverse set of institutions

  Gaetano Borriello, Donna Brown, Enoch Hwang, A. Scottedward Hodel, Joseph Hughes, Dean Johnson, Eric Johnson, Charles Kime, Randy Katz, David Livingston, Afsaneh Minaie, Kevin Nickels, Mohamed Rafiquzzaman, Frank Vahid, Ramachandran Vaidyanathan, Zvonko Vranesic, John Wakerly, Nancy Warter, and Sally Wood.
- **4 phase process**
  – Conducted anonymously
    - To prevent reputation from swaying people

Phase 1: Identifying the Concepts

- Experts suggested wide range of concepts
- 2-3 researchers independently coded, clustered suggestions
- Researchers reconciled shortened lists



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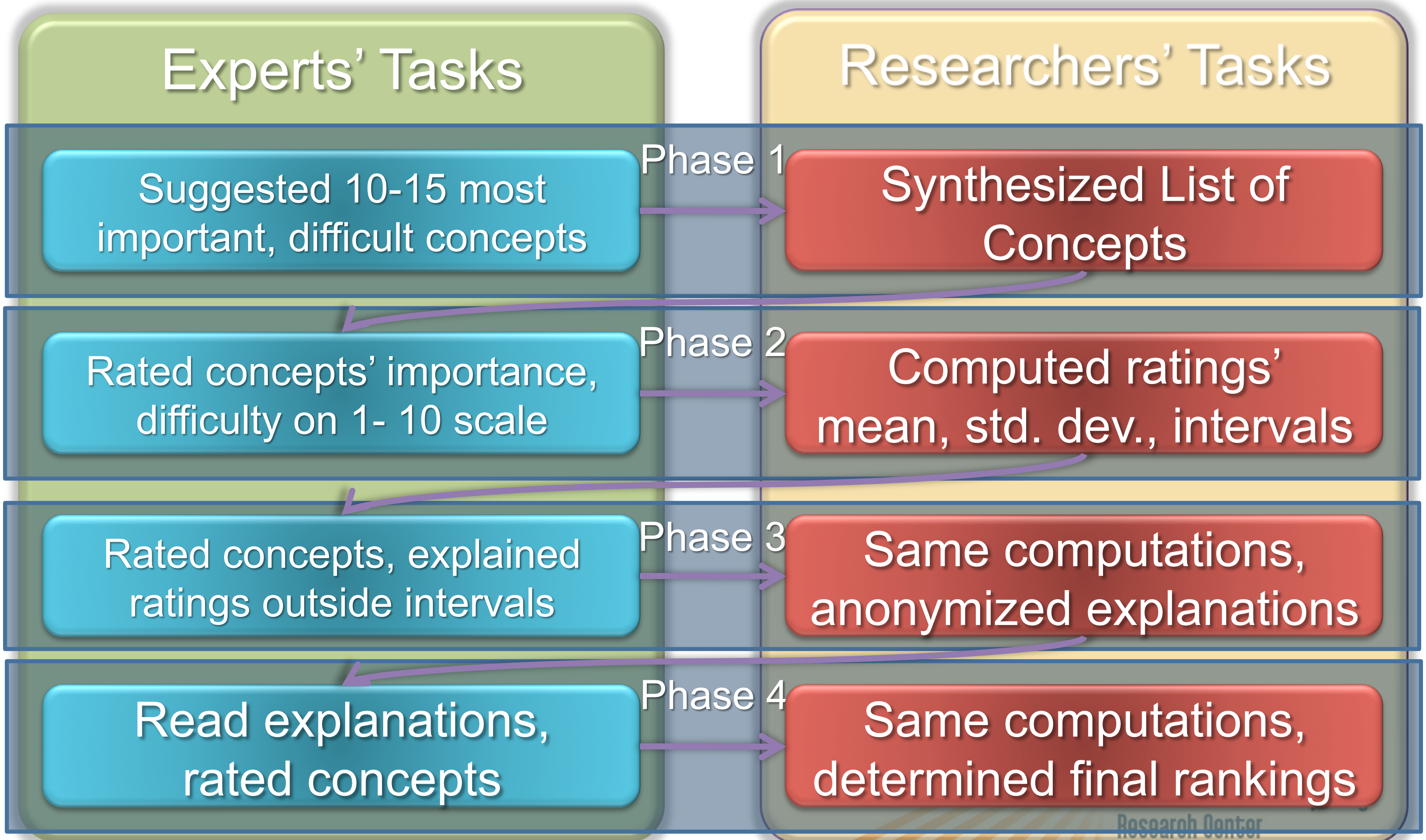
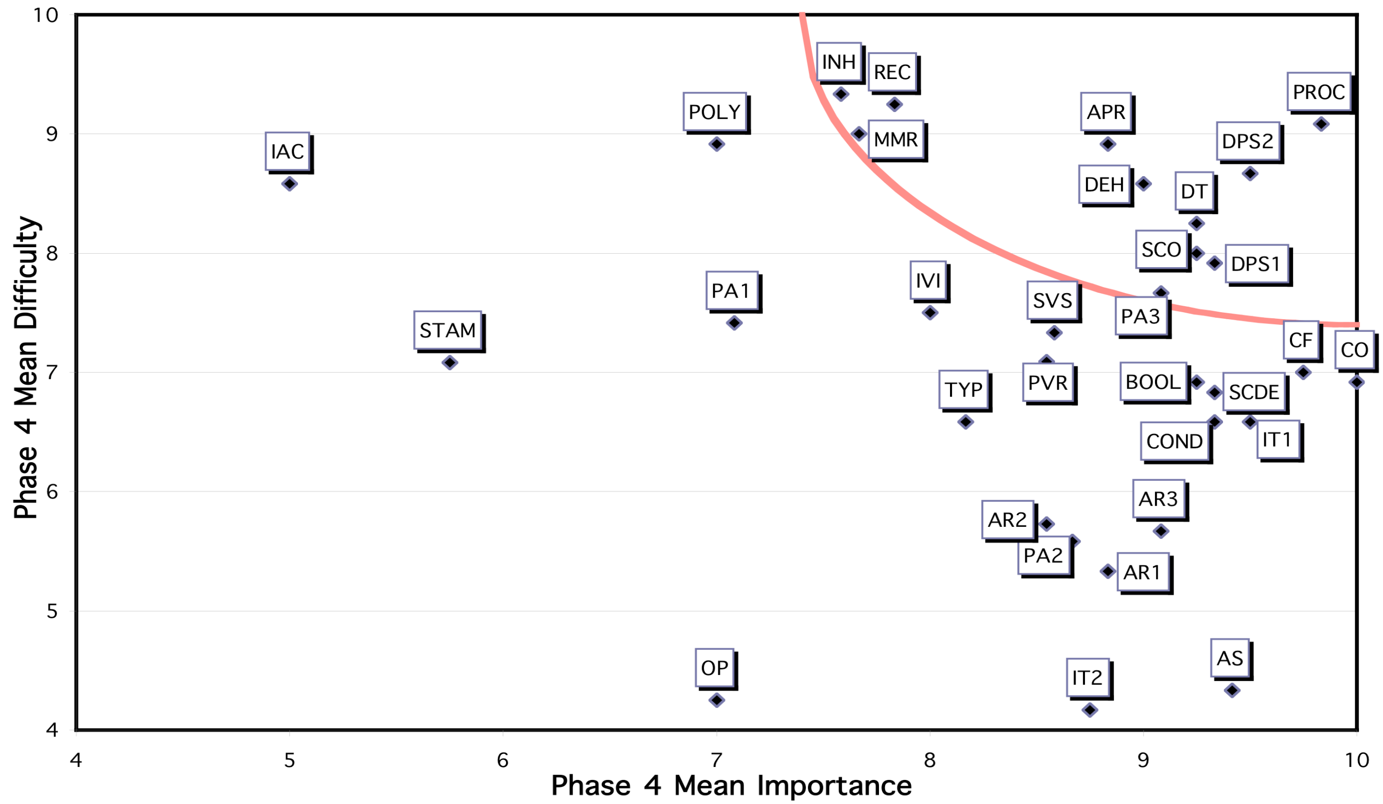
Phases 2-4: Rating Process



What we found



Data from the CS1 delphi process

- An "important and difficult" subset

Digital Logic Consensus

- Understand, manipulate state in different contexts
  - 4 of 11 most important and difficult
- Divide and conquer design problems
  - 4 of 11 most important and difficult

Top 11 Most Important and Difficult Concepts
Converting Verbal Specs to State Diagrams
Timing Diagrams to State Machines
Debugging, Troubleshooting and Designing Simulations
Converting Algorithms to RTL
State Transitions
Converting Verbal Specs to Boolean Logic
Designing Control for Datapaths
Modular Design
Multilevel Synthesis
Hierarchical Design
Sequential Circuit Corresponds to State Diagram

Step 2

- **Identify Student Misconceptions**
  - Go to the source: the students

Methodology

- **Have students solve difficult problems**
  - While verbalizing their thought process
  - 7 B/C-level students (2 women/5 men, 2 inter)
- **Record / Transcribe**
- **Qualitative Analysis:**
  - Researchers “code” interviews independently
  - Reconcile coding
  - Identify themes

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Lessons Learned (so far)

- **A sampling from two publications:**

Student Misconceptions in an Introductory Logic Design Course [ASEE 2006]

Proof by Incomplete Enumeration and Other Logical Misconceptions [ICER 2008]

Converting English to Boolean

A campus sandwich shop has the following rules for making a good sandwich:

(1) **h + r + t (OR)** must have at least one type of meat,

(2) **hr' + rh' (XOR)** must have roast beef or ham, but not both,

(3) **t' + c (implication)** has turkey then it must also have cheese.

Write a Boolean expression for the allowed combinations of sandwich ingredients using the following variables:

c = cheese *h = ham* *r = roast beef* *t = turkey*

(h + r + t)(hr' + rh')(t' + c) -> (hr' + rh')(t' + c)

Easy and Hard Operators

- **Easy operators: AND, OR, XOR**
  - Correct routinely, could explain why
- **Hard operators: if-then (implication), if-and-only-if (XNOR), not both (NAND)**
  - Frequently incorrect, false reasoning
  - Failure to draw truth tables
  - Often “simplified” to easier concepts

Reduction to Easy Operators

inputs		*easy operators*					
A	**B**	NAND	XOR	OR	AND	if A then B	if-and-only-if
0	**0**	1	0	0	0	1	1
0	**1**	1	1	1	0	1	0
1	**0**	1	1	1	0	0	0
1	**1**	0	0	1	1	1	1

- **Proof-by-incomplete enumeration**
- **Forgetting negated cases**

Examples

INTERVIEWER: So how'd you come up with (a'c OR ac') for "do not use both?"

STUDENT 2: Well, when we do not have all spice, I mean it says do not use all spice and nutmeg simultaneously (<a,c> = <1,1>), right?

INTERVIEWER: Ok.

STUDENT 2: So if allspice is not being used, we can use cinnamon (<a,c> =<0,1>). And if allspice is used, then we cannot use cinnamon (<a,c> = <1,0>).

STUDENT 5: If you have turkey, then you must also have cheese [writes +tc] so it's **turkey AND cheese**

STUDENT 3: And then [rule] 3... I guess would just be like, turkey implies cheese, so let's see... **turkey AND cheese** (<t,c> = <1,1>) because... **OR.. NOT turkey AND cheese** (<t,c> = <0,1>)? I think, because this would be such true, if it has turkey and cheese, but it doesn't say anywhere that cheese cannot be by itself. So this can also be true. [**writes tc + t'c**].

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Omission of Complemented Variables

- **Failed to enumerate cases (as above)**
- **Left them out of expressions:**
  - A sandwich with just cheese written as **c**
    - Instead of **ch’t’r’**

STUDENT 5: You can use cinnamon by itself without the nutmeg, because that doesn't break rule (2) [writes **+c**] … or you could just use allspice by itself [writes **+a**].

Others

- **Lack of meta-cognition**
  - Never return to original specification
- **Cowboy composition & non-systematic approaches**
- **If-then misconceptions**
  - False Antecedent confusion
- **Recall vs. reasoning**

State Encoding

A state diagram with n states requires at least m flip-flops to implement a sequential circuit. If a different state diagram has $2n$ states, what is the minimum number of flip-flops needed for an implementation?

a.) m

b.) $m + 1$

c.) $2m$

d.) $2m + 1$

e.) m^2

f.) $m^2 + 1$

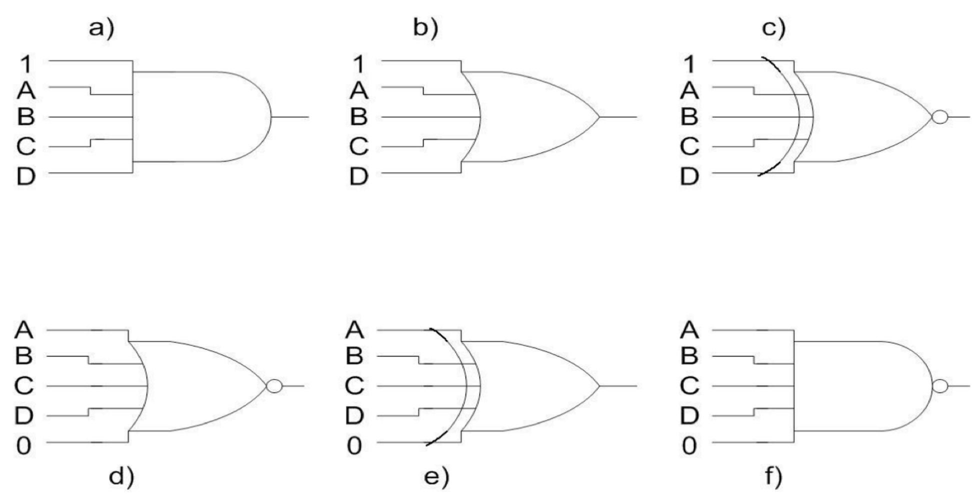
g.) None of the above

- 65% correct

Gate-level Reasoning

Which of the following will result in nontrivial output (not always 0 or 1)?

Select all correct answers.



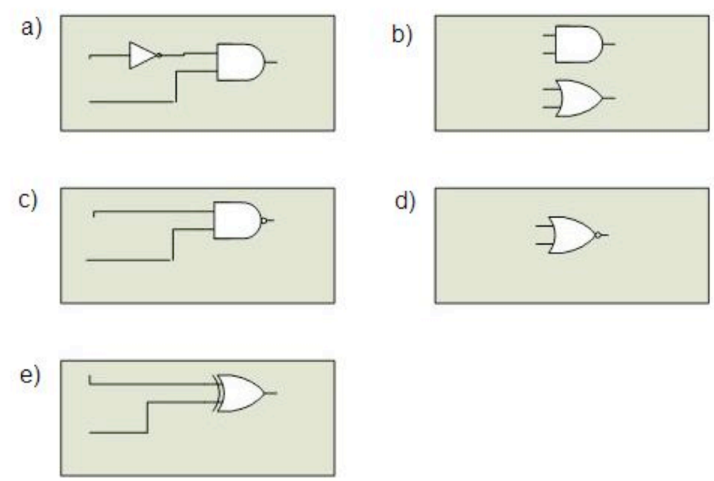
g.) None of the above

- 59% correct

Logical Completeness

Which of the following are complete logic families (i.e., all possible Boolean functions can be implemented using just these gates and the constants 0 and 1).

Select all correct answers.



f.) None of the above

- 16% correct

Future Steps

- Step 3: Develop Concept Inventory
- Step 4: Validate
- Step 5: Disseminate

Summary

- **Significant steps forward in teaching** requires scientific approach to pedagogy
  - **Develop tools to measure learning**
- **Our efforts based on previous success in another discipline (Physics)**
- **Big project, making steady progress**
  - **But, our findings can guide teaching now**

http://www-faculty.cs.uiuc.edu/~zilles/csci.html

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